Two-stage betatron and momentum collimation studies with applications to LHC

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We consider a single passage of the circulating beam through some set of collimators, primary and secondary ones, all located in a bend free lattice section. We assume that the primary collimators are "pure scatters" – circulating particles scatter at their edges (and only there) in forward direction thus creating secondary halo. The function of the secondary collimators positioned at a little higher aperture n_2 is to intercept the halo. The secondaries are "black absorbers" – any particle touching them is considered lost.

In this report we investigate the following problems [1], [2]:

Let's fix the lattice of the collimation section and the collimators. How to compute the maximum betatron invariants (amplitudes) of a secondary halo particle surviving all secondary collimators: in-plane horizontal $A_{x,max} \equiv max\sqrt{x_0^2 + x_0'^2}$, vertical $A_{y,max} \equiv max\sqrt{y_0^2 + y_0'^2}$ and combined $A_{max} \equiv max\sqrt{x_0^2 + x_0'^2 + y_0^2 + y_0'^2}$. In these definitions x_0, y_0 are initial normalized coordinates of halo particles (points along the primary collimator borders) and x_0', y_0' are initial normalized angles ranging from $-\infty$ to $+\infty$ to describe all possible forward directions.

For arbitrary optics and set of halo sources x_0, y_0 and for the case of flat collimators (pairs of opposing jaws), $A_{x,max}, A_{y,max}, A_{max}$ are computed by the code DJ (Distribution of Jaws) [3]. Circular collimators can be approximated by many-side polygons and treated the same way. The mapping procedure implemented in DJ is equivalent to linear tracking but is much faster.

To describe momentum collimation in an arbitrary lattice (matched dispersion non-zero) we consider [4] monochromatic δ-fraction of the circulating beam and assume that the relative off-momentum offset $\delta \equiv \Delta p/p$ of the particles is the same before and after scattering. The same mapping technique is then used to compute the momentum dependence of the maximum *betatronic* parts of the amplitudes of escaping particles: $A_{x,max}(\delta)$, $A_{y,max}(\delta)$, $A_{max}(\delta)$. These functions are decreasing in most cases and continuous if the set of source points is continuous. Some restrictions apply on the value of the normalized dispersion at the primary collimator and its derivative, if one wants to protect over one turn locations with highest dispersion around the ring (the LHC arc).

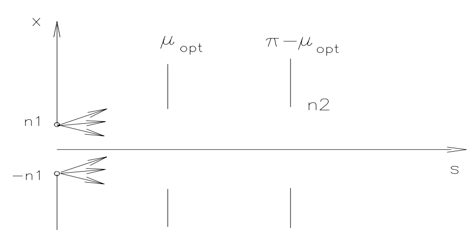
For betatron collimation, the combined invariant A_{max} (important for the LHC) can be computed without a code, by plotting contours of constant 4D-emittance on the plane of initial betatron phases. In a simple example (lattice) we show that small A_{max} can only be obtained if the difference between horizontal and vertical betatron phase advances (the split) varies along the collimation section. The role of the varying phase advance split was first pointed out by Risselada [5].

Next comes the optimization problem: where and what secondary collimators one should locate to get minimum $A_{x,max}$, $A_{y,max}$, A_{max} for a given δ (or a combination of these taken with some weights)? The code provides automatic minimization procedure (simulated annealing method) which has the advantage that it produces many equivalent solutions (secondary jaw distributions).

1 2-STAGE BETATRON COLLIMATION SYSTEMS

1.1 1D case

In case of collimation in a plane, Fig 1, the primary collimator is located at $s = 0$ at amplitude n_1 (in units of σ) and the secondary ones are at amplitude $n_2 > n_1$. The phase advance along the beamline is $\mu(s)$, with $\mu(0) = 0$. The halo sources are two points $x_0 = \pm n_1$ with initial non-normalized angles within $(-\frac{\pi}{2}, \frac{\pi}{2})$. The task is to find secondary jaw configuration that minimizes $A_{max} \equiv max\sqrt{x_0^2 + x_0'^2}$ (maximum is taken over all normalized angles x_0' corresponding to surviving trajectories). The optimum configuration is: two secondary collimators located at phases μ_{opt} and $\pi - \mu_{opt}$, where $\mu_{opt} \equiv arccos(n_1/n_2)$. The minimum value is $A_{max} = n_2$ and at optimum the maximum initial angle corresponding to surviving trajectory is $|x'_{0,max}| = \sqrt{n_2^2 - n_1^2}$. We notice that $A_{max} = n_2$ is the smallest achievable amplitude, i.e. the system acts as a *"1 D pipe"*.



1.2 2D case, circular symmetry

All primary and secondary collimators are circular and the horizontal and vertical phase advances are equal $\mu_x(s) = \mu_y(s)$ for all s. The halo is defined by: $x_0^2 + y_0^2 = n_1^2$ with non-normalized angles in both planes within $(-\frac{\pi}{2}, \frac{\pi}{2})$. The aim is to minimize $A_{max} \equiv max\sqrt{x_0^2 + x_0'^2 + y_0^2 + y_0'^2}$. The optimum phases are $\mu_{x,opt}$, $\pi - \mu_{y,opt}$ and $\pi/2$. Using the two optimum collimators described above 1D pipe is created in each x and y plane, but also a third collimator at phase $\pi/2$ has to be added in order to minimize A_{max}. For instance with $n_1 = 6, n_2 = 7$, one gets $A_{max} \sim 9.2$.

1.3 2D case, arbitrary lattice

Now let the lattice is described by two arbitrary phase advance functions $\mu_x(s), \mu_y(s)$ with $\mu_x(0) = \mu_y(0) = 0$ and the halo sources and minimized object A_{max} are the same as in the previous section. Equivalently, we can describe the optics by the "split function" $\mu^-(\mu^+)$ where $\mu^\pm(s) \equiv \mu_x(s) \pm \mu_y(s)$. What is the optimum arrangement of secondary collimators?

– it can be found numerically for the case of jaws. The algorithm is given in the next chapter for the more general case of momentum collimation (arbitrary δ).

– a "2D pipe" can be built for every source point ($x_0,\ y_0$) in the following way. One should place a collimator for every solution (pair $\mu_x,\ \mu_y$) of the equation:

$$\frac{(x_0)^2}{\cos^2\mu_x} + \frac{(y_0)^2}{\cos^2\mu_y} = n_2^2 \tag{1}$$

If so, then the absolute minimum $A_{max} = n_2$ is achieved.

This definition may look too abstract because 1) even for one source point it requires an infinite number of collimators and 2) if x_0,y_0 are unequal and both non-zero then realistic lattices satisfying it do not exist. We expect however that for solutions providing $A_{max} \sim n_2$, i.e for a large number of optimized collimators in a "good" lattice, the optimum set of phases is not far from the ideal one. It can also be demonstrated that in order to minimize A_{max} lattice locations s with both large and small split $\mu^-(s)$ must be present. The proof is based on the "method of contours" – an alternative method to compute A_{max} without any code. We postpone with this until Chapter 3.

2 OPTIMIZATION OF A JAW COLLIMATION SYSTEMS (BETATRON AND MOMENTUM) BY CODE DJ (DISTRIBUTION OF JAWS).

2.1 Halo definition for $\delta \neq 0$

To define the halo we follow the motion of some circulating particle with $\delta \neq 0$, which becomes particle of the halo trough scattering at point $P = (x_0, y_0)$ (angle and amplitude jump).

The horizontal invariant before the scatter (Fig 1) is $A_{x,circ\ beam} = |x_0 - \delta\ \eta_0|$ and after the scatter the invariants are: horizontal

$$A_x(x_0, x_0') = \sqrt{(x_0 - \delta\eta_0)^2 + (x_0' - \delta\eta_0')^2} \tag{2}$$

and combined

$$A(x_0, x_0') = \sqrt{(x_0 - \delta\eta_0)^2 + (x_0' - \delta\eta_0')^2 + y_0^2 + y_0'^2} \tag{3}$$

Within the collimation section (there are no bends; the radius vectors of **3** and **4** perform rigid rotation around the origin) the halo trajectory **3** is:

$$x(s) = x_0\ \cos\mu_x(s) + x_0'\ \sin\mu_x(s) \tag{4}$$

(+ similar for y). Here $\eta \equiv \frac{D_x}{\sqrt{\beta_x\epsilon}}$ is the normalized dispersion $D_x \neq 0$ is the matched dispersion, ϵ is the emittance. Everywhere index 0 denotes values at the primary collimator (point P) and η' denotes derivative of η with respect to the horizontal phase advance μ_x.

In the LHC case, with one pair of opposing primary jaws, Fig 1, the halo exists ($A_{x,circ\ beam} \neq 0$) if 1) $|x_0| \geq |\delta\eta_0|$ and 2) x_0 has the same sign as $\delta\eta_0$ (the circle should not intersect the opposing jaw). Only those sources x_0, y_0 contribute to the δ-fraction of the halo for which 1) and 2) are simultaneously fulfilled. If η_0 is positive, then the halo momenta satisfy: $|\delta| < \delta_{max} \equiv \max_P |x_0|/\eta_0$, where $\max_P$ means maximum over the sources P.

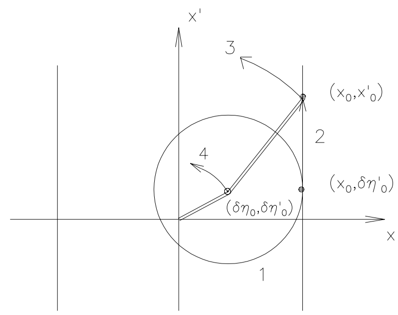


Figure 1: **1** Circulating beam particle invariant circle. **2** Scattering. **3** Secondary halo trajectory. **4** δ-centre motion.

2.2 Halo Computation for $\delta \neq 0$ (code DJ)

Each pair of opposing parallel jaws is defined by its longitudinal coordinate s and tilt angle α around the longitudinal axis. For fixed jaws, lattice functions $\mu_x(s)$, $\mu_y(s)$, $\eta(s)$ and δ, the maximum amplitudes $A_{x,max}$, $A_{y,max}$, A_{max} surviving all secondary jaws are computed as follows:

1. *Generate* N_P points $P = (x_0, y_0)$ along primary jaw borders;

2. For each point P:

– *map* (Fig 2) the line boundaries of all secondary jaws on the plane of initial-angles (x_0', y_0') by using the reverse of (4) (δ-independent linear transform)

– *find* all intersecting points of line-images; among these points find the vertices $(x_0', y_0')_i$ ($i = 1...N_{vert}^{(P)}$) of the *"escape window (a polygon)"*.

– *compute* at each vertex $A_{x,i} = A_x(x_0, x_{0,i}')$

– find the largest $A_{x,i}$: $A_{x,max,P} = \max_i\ A_{x,i}$

3. Repeat the same for all points P and compute the maximum of the maxima:

$A_{x,max} = \max_P A_{x,max,P};\ P \in all\ primary\ POJ$

DJ also stores the maximum initial angle corresponding to surviving trajectory and the vertex index corresponding to $A_{x,max}$ (maximum vertex). The same is done for A_{max} and $A_{y,max}$.

The escape polygon $(x_{0,i}', y_{0,i}')$ is independent of δ. It depends only on the lattice and the jaw setup, and represents an escape window in angle space, whose corners move, or may be screened out, as the source P is varied.

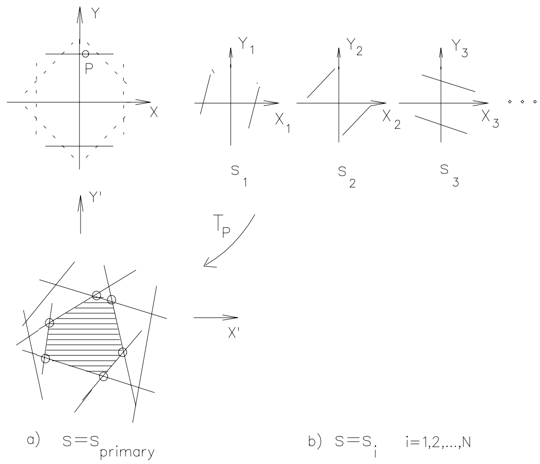


Figure 2: (a) - Normalized coordinate space (above) and angle space (below) at the longitudinal position $s_{primary}$ of the primary pair of jaws; (b) N secondary pairs of jaws. For each point P: 1) each pair of parallel lines (stripe) in coordinate space is mapped into a stripe in angle space; 2) the overlap region of all stripes forms the escape polygon (shaded).

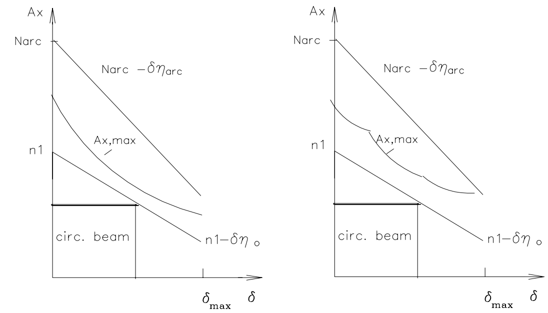


Figure 3: δ dependent halo limits

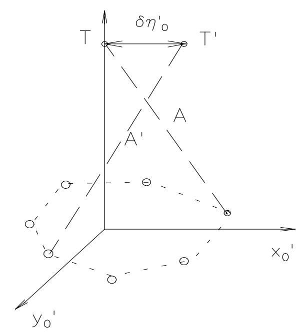


Figure 4: Escape polygon and maximum vertex

2.3 δ-dependence of the halo limits. The requirement for zero derivative of the normalized dispersion at the primary collimator.

In the LHC only horizontal primary jaws are present ($|x_0| = n_1$), therefore after minimization the source $P = (x_0, y_0)$ producing A_{max} is for all δ the outermost point of the jaw (the one with maximum y_0). One can therefore assume for a while that P is fixed.

We represent the surviving halo on the plane amplitude-δ, for simplicity taking only the fraction with $\delta > 0$. The invariants of surviving particles are below the curve $A_{max}(\delta)$. This curve is continuous if the sources are, although its derivative may not be.

If the maximum vertex (x'_{0,i_A}, y'_{0,i_A}) is fixed the (explicit) dependence on δ is given by:

$$A_{max} = \sqrt{(x_0 - \delta\eta_0)^2 + (x'_{0,i_A} - \delta\eta'_0)^2 + y_0^2 + y'^2_{0,i_A}} \quad (5)$$

If the normalized dispersion derivative at the primary collimator $|\eta'_0|$ is zero, then the maximum vertex index does not change and A_{max} is given by (5) for all δ – monotonously decreasing function, Fig 3 (left). If η'_0 is nonzero, then $A_{max}(\delta)$ is composed of mini-curves of the kind (5), each corresponding to a new maximum vertex, Fig 3 (right). In this case $A_{max}(\delta)$ is still decreasing, but not as fast. From this point of view *having small $|\eta'_0|$ is preferred, but not an absolute condition for momentum collimation.*

Fig 3 shows the vertices of the escape polygon $(x'_{0,i}, y'_{0,i})$ and $A \equiv A_{max}$, which is defined as the maximum distance from the point T to a vertex. As δ increases, T advances to T′, the maximum-vertex changes, but A_{max} is continuous (A=A′). In the case $\eta'_0 = 0$, T remains on the ordinate axis and the maximum vertex is independent of δ.

Another advantage of placing the primary collimator at a location with $\eta'_0 = 0$ is that *minimization of A_{max} can be done with equal results for any δ.* This is because both the escaping polygon and the index i_A are δ-independent. An optimum jaw arrangement found for $\delta = 0$ (betatron collimation) remains the optimum one for any δ.

If the set of sources P is arbitrary (skew primary also present) the maximum source P may also change with δ. In general $A_{max}(\delta)$ is built of mini-curves (5) each corresponding to a different value of x_0, y_0 and/or of the maximum vertex i_A.

2.4 Protecting ring locations with maximum dispersion. The requirement for high normalized dispersion at the primary collimator.

We denote by N_{arc} (σ_x units) the available horizontal aperture at a ring location where the dispersion D_x is maximum. At this location $\eta_x = \eta_{arc}$ and the halo particle coordinate (4) is:

$$x_{arc} = A_x \cos(\mu_{x,arc} + \mu_0) + \delta\,\eta_{arc} \leq A_x + \delta\eta_{arc} \quad (6)$$

On Fig 3, left and right, the secondary halo occupies an area restricted from below by the line representing the primary collimator (with slope η_0) and from above by the curve $A_{x,max}(\delta)$. For the arc to be protected ($x_{arc} < N_{arc}$) the latter curve should be below the line representing the arc (slope η_{arc}):

$$A_{x,max} < N_{arc} - \delta\eta_{arc} \quad (7)$$

A large normalized dispersion at the primary $\eta_0 \sim \eta_{arc}$ (nearly parallel lines) is needed because it provides wider amplitude interval for the halo at δ near δ_{max}.

For example we take as before $x_0 = n_1$, $\eta_0' = 0$ and assume that a 1D pipe is created in the horizontal plane. The value of the maximum vertex is known: $x'_{0,i_{Ax}} = \sqrt{n_2^2 - n_1^2}$ and we can use it to compute (7) for $\delta = \delta_{max} = x_0/\eta_0$. In the expression for $A_{x,max}$ ((5) without the y terms) the first term under the square root disappears and we get $A_{x,max} = x'_{0,i_{Ax}} = \sqrt{n_2^2 - n_1^2}$. The condition for protected arc is:

$$\sqrt{n_2^2 - n_1^2} + (\eta_{arc}/\eta_0)\, n_1 < N_{arc}. \tag{8}$$

To improve things one can decrease either the ratio η_{arc}/η_0, or both n_1 and n_2. The latter choice may cause the primary collimator to cut into the circulating beam rectangle near the bucket edge, Fig 3. The vertical size of this rectangle is defined by the betatron primaries.

With LHC parameters: $n_1 = 8$, $n_2 = 9$, $\eta_{arc} = 0.16/\sqrt{\epsilon_x}$, $\eta_0 = 0.2/\sqrt{\epsilon_x}$ the condition (8) becomes $N_{arc} > 10.5$.

3 SIMPLE BETATRON COLLIMATION THEORY WITH PHASE ADVANCE SPLIT

3.1 The halo emittance function $\mathcal{A}$ and the collimator function $\mathcal{C}$

Halo is defined by initials: x_0, y_0 on the circle $x_0^2+y_0^2 = n_1^2$ and normalized angles $-\infty < x_0' < \infty$,$-\infty < y_0' < \infty$. We introduce initial phases μ_{x0}, μ_{y0} as shown on Fig 5.

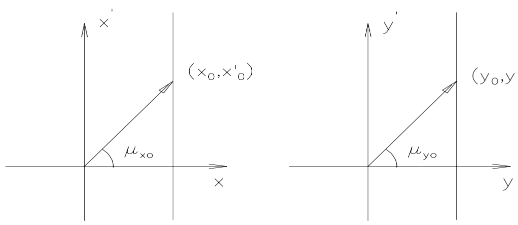


Figure 5: Definition of μ_{x0}, μ_{y0}.

The halo trajectories are

$$x = A_{x0} \cos(\mu_{x0} - \mu_x), \tag{9}$$

where $A_{x0} = \sqrt{x_0^2 + x_0'^2} = x_0/\cos\mu_{x0}$ (similar for y). The four dimensional emittance of particles in the halo produced by the source x_0, y_0 is $A = \sqrt{A_{x0}^2 + A_{y0}^2}$ so we denote $\mathcal{A} \equiv (A/n_2)^2$:

$$\mathcal{A}(\mu_{x0}, \mu_{y0}) = \frac{(x_0/n_2)^2}{\cos^2\mu_{x0}} + \frac{(y_0/n_2)^2}{\cos^2\mu_{y0}} \tag{10}$$

$\mathcal{A}$ as always positive and reaches a minimum for $\mu_{x0} = 0, \mu_{y0} = 0$, with value $n_1^2/n_2^2 < 1$. $\mathcal{A}$ has periodicities π

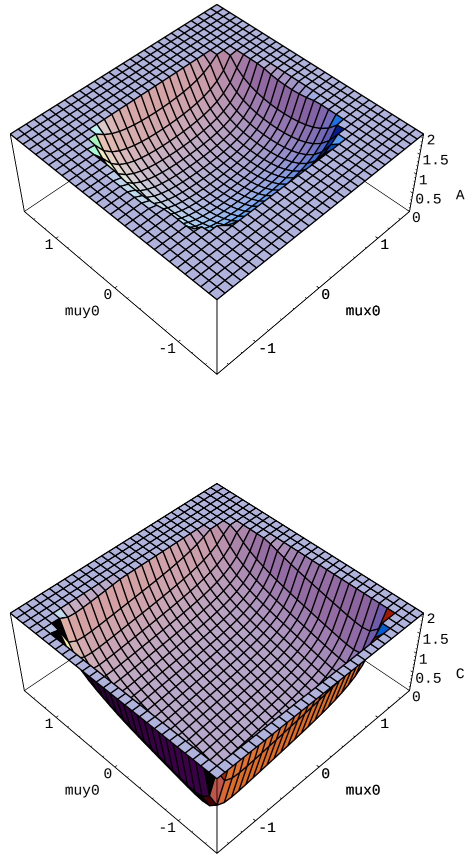


Figure 6: Functions $\mathcal{A}(\mu_{x0}, \mu_{y0})$ (top) and $\mathcal{C}(\mu_{x0}, \mu_{y0}, 0.535, 0.535)$ (bottom) on the square $-\frac{\pi}{2} < \mu_{x0}, \mu_{y0} < \frac{\pi}{2}$. $\mathcal{C}$ is below $\mathcal{A}$ except at the collimator point where they are tangent

in both coordinates. The surface $\mathcal{A}$ looks like a bowl that is asymptotic to a square chimney, Fig 6 (top).

Consider a secondary collimator located at phase advances μ_x, μ_y. At such collimator:

$$x = A_{x0}\cos(\mu_{x0} - \mu_x);\; y = A_{y0}\cos(\mu_{y0} - \mu_y) \tag{11}$$

All particles for which $(x^2 + y^2)/n_2^2 > 1$ are stopped, so we define the secondary collimator function $\mathcal{C}$

$$\mathcal{C}(\mu_{x0}, \mu_{y0}, \mu_x, \mu_y) = \tag{12}$$
$$\frac{(x_0/n_2)^2}{\cos^2\mu_{x0}}\cos^2(\mu_{x0} - \mu_x) + \frac{(y_0/n_2)^2}{\cos^2\mu_{y0}}\cos^2(\mu_{y0} - \mu_y)$$

Particles with $\mathcal{C} > 1$ are stopped. $\mathcal{C}$ has the same periodicity and same asymptotes as $\mathcal{A}$ so we may compare them directly. The collimator function $\mathcal{C}$, Fig 6 (bottom), is everywhere below $\mathcal{A}$ except at the point $\mu_{x0} = \mu_x,\ \mu_{y0} = \mu_y$, *where they are tangent*.

3.2 Betatron collimation analysis

For example, we take some source point $x_0/n_2 = 0.5, y_0/n_2 = 0.7$ $(n_1/n_2 = 0.86)$ and locate in the lattice a single circular collimator at phases $\mu_x = \mu_y = \mu_o$, where $\mu_0 = 0.535$ is the solution of $\mathcal{A}(\mu_o, \mu_o)$=1), hence the collimator pair phases is on the contour $\mathcal{A}$=1 (the reason will become clear soon)

Fig 7 shows several contours of $\mathcal{A}$ and the contour $\mathcal{C}(\mu_{x0}, \mu_{y0}, 0.535, 0.535)$ =1. The normalized squared A_{max} is simply the maximum $\mathcal{A}$-contour value that can be found within the "escape window" which is the inside portion of the contour $\mathcal{C}$=1, Fig 8.

Now we take four collimators (Fig 9) at phases:
1: (μ_o, μ_o) , split = 0
2: $(-\mu_o, -\mu_o)$, split = 0
3: $(\mu_o, -\mu_o)$, split = 2μ_o
4: $(-\mu_o, +\mu_o)$, split = -2μ_o
The intersection of the inside portion of all four contours gives the set of particles escaping the system. The maximum "escaping" $\mathcal{A}$ is 1.3, so $A_{max} = \sqrt{1.3}\, n_2$ = 1.14 n_2

The surfaces $\mathcal{A}$ and $\mathcal{C}$ are tangent at the collimator point. Therefore large A_{max} cannot escape if the collimator phases are chosen near the contour $\mathcal{A}$=1 and this contour is surrounded by collimator contours from all sides. The last condition is only possible if *some of the collimators are at locations with large enough split between the horizontal and vertical phase advances*. If there is collimator for each solution of $\mathcal{A}(\mu_x, \mu_y) = 1$, then all particles with $\mathcal{A}_{max} > 1$ will be stopped (2D pipe).

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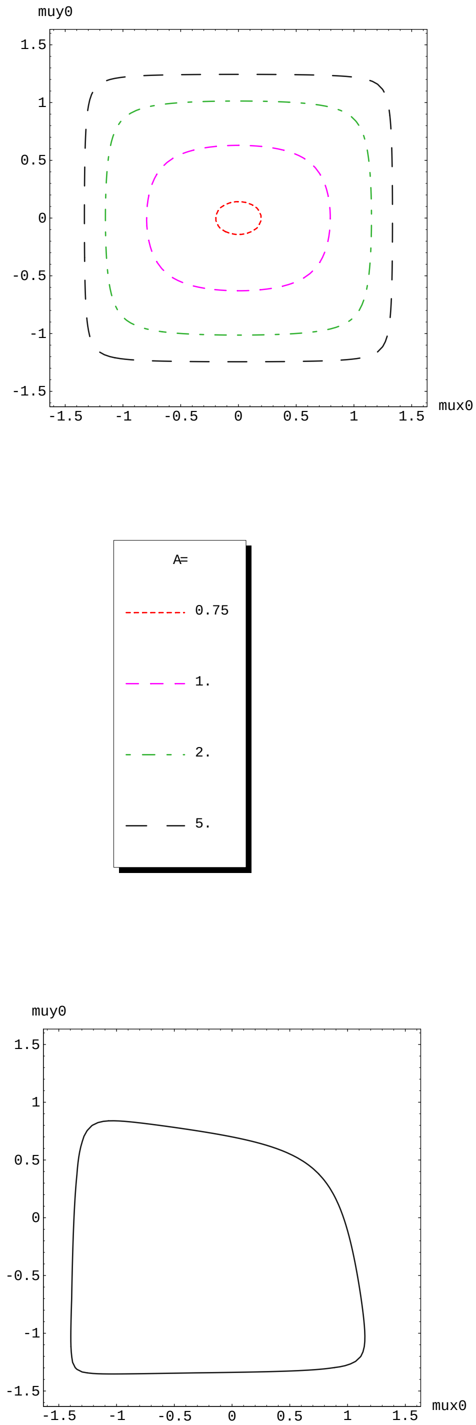

Figure 7: Contours A=0.75, 1, 2, 5 (top) and contour C=1 (bottom).

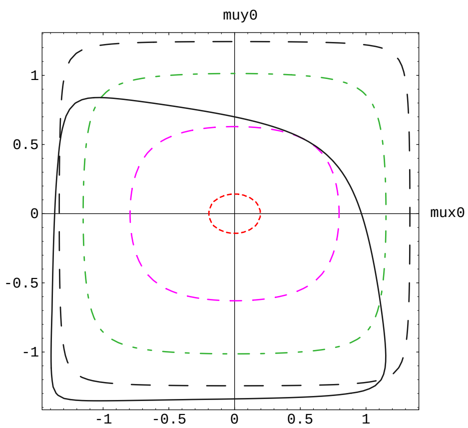

Figure 8: One collimator with betatron phases chosen on the contour $\mathcal{A}$=1.

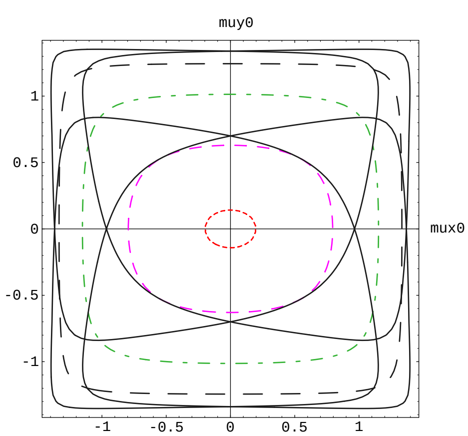

Figure 9: Four collimators with phases on the contour $\mathcal{A}$=1.