

# General Beam-beam detuning formula and its tests

## TRI-BN 2220

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**ARTICLE HISTORY**  
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### 1. Introduction

In previous reports [1], [2] an analytic formula describing the amplitude-dependant tune shifts  $\Delta Q_{x,y}$  (detuning) generated by an individual long-range (l.r.) collision in the LHC was found to agree with the detuning found with MadX. This formula, first derived in [2], is valid only in the case of an exact anti-symmetry between the weak and strong beam rms sizes at the collision location:

$$\sigma_x^{\text{str}} = \sigma_y^{\text{wk}}, \quad \sigma_y^{\text{str}} = \sigma_x^{\text{wk}} \quad (1.1)$$

It provides a recipe to compute the two (horizontal, vertical) detunings:

$$\begin{aligned} \Delta Q_x(a_x, a_y; \theta) \\ \Delta Q_y(a_x, a_y; \theta) \end{aligned} \quad (1.2)$$

as a function of an array of optics parameters at the l.r. location  $\theta = (d_x, d_y, r)$ , defined in the next section. Only normalized parameters participate in  $\theta$ . The formula was encoded in *Mathematica* [1], [2] and python.

In this paper Eqn 1.2 is modified so as it is valid also in case of a general collision, i.e. when

$$\sigma_x^{\text{str}} \neq \sigma_y^{\text{wk}}, \quad \sigma_y^{\text{str}} \neq \sigma_x^{\text{wk}} \quad (1.3)$$

The result found is as follows. With

$$A \equiv \frac{\sigma_x^{\text{wk}}}{\sigma_y^{\text{str}}}, \quad B \equiv \frac{\sigma_y^{\text{wk}}}{\sigma_x^{\text{str}}}. \quad (1.4)$$

the modified expressions 1.2, i.e. a general l.r. detuning formula is

$$\begin{aligned} A^2 \Delta Q_x(A a_x, B a_y; \theta') \\ B^2 \Delta Q_y(A a_x, B a_y; \theta') \end{aligned} \quad (1.5)$$

where  $\theta'$  now contains the two additional parameters  $\sigma_{x,y}^{\text{wk}}$ .

Eqn 1.5 is here tested on a simple a MadX model that consists in some ring lattice counting a single l.r. collision.

## 2. Notes on derivation

The detunings are derivatives of the averaged Hamiltonian, i.e. the zero-index coefficient in its decompositions as Fourier series, [2]:

$$\Delta Q_z(a_z; \theta) = \frac{2\xi}{a_z} \frac{\partial C_{00}(a_z; \theta)}{\partial a_z}, \quad \text{where } z = x \text{ or } y \quad (2.6)$$

( $\xi \equiv \frac{N_b r_0}{4\pi\gamma\epsilon}$  is the beam-beam parameter). The Hamiltonian is the one that governs the Bassetti-Erskine kick:

$$\begin{aligned} H(x, y; \theta) &= \int_0^1 \frac{dt}{t g_r(t)} [1 - e^{-t(P_x + P_y)}], \\ P_x &\equiv \frac{1}{2(\sigma_x^{\text{str}})^2} (x + \mathfrak{D}_x)^2, & P_y &\equiv \frac{1}{2g_r(t)^2 (\sigma_x^{\text{str}})^2} (y + \mathfrak{D}_y)^2; \\ g_r(t) &\equiv \sqrt{1 + (r^2 - 1)t}, \end{aligned} \quad (2.7)$$

where  $r \equiv \frac{\sigma_y^{\text{str}}}{\sigma_x^{\text{str}}}$  is the strong-beam sigma aspect ratio at this location. Also  $\mathfrak{D}_x = x_{\text{CO}}^{\text{wk}} - x_{\text{CO}}^{\text{str}}$  is the full separation – a difference between closed orbit offsets, and similar for  $\mathfrak{D}_y$ .

As a side note, it can be shown that an exchange of the all subscripts  $x$  and  $y$ , i.e. horizontal and vertical axes, preserves  $H$ , and this allows to derive expressions for IR1 from the ones in IR5, or reverse.

By definition of amplitudes  $a_{x,y}$ :

$$\begin{aligned} x &= \sqrt{2\beta_x^{\text{wk}} J_x} \sin \phi_x = a_x \sigma_x^{\text{wk}} \sin \phi_x \\ y &= \sqrt{2\beta_y^{\text{wk}} J_y} \sin \phi_y = a_y \sigma_y^{\text{wk}} \sin \phi_y, \end{aligned} \quad (2.8)$$

and using the symmetry Eqns 1.1 we get

$$\begin{aligned} x &= r a_x \sigma_x^{\text{str}} \sin \phi_x \\ y &= a_y \sigma_x^{\text{str}} \sin \phi_y, \end{aligned} \quad (2.9)$$

where  $\beta_{x,y}^{wk}$  are the weak-beam beta-functions  $\sigma_{x,y}^{wk} \equiv \sqrt{\epsilon \beta_{x,y}^{wk}}$  and  $a_{x,y}$  are the horizontal and vertical amplitudes.

Clearly, when  $x,y$  from 2.9 are substituted in  $H$ , then  $\theta$  contains only normalized (to sigmas) separations:  $d_x = \frac{\mathcal{D}_x}{\sigma_x^{str}}$ ,  $d_y = \frac{\mathcal{D}_y}{\sigma_y^{str}}$  and  $r$ . Note that  $\theta$  can be chosen either  $\theta = (d_x, d_y, r)$  (normalized to strong-beam sigma full separations and the aspect ratio), or  $\theta = (d_x/r, rd_y, r)$  i.e. via normalized to weak-beam sigma separations.

The desired transforming coefficients  $A$  and  $B$ , Eqn 1.4, are the ratios between the right-hand sides of Eqns 2.8 and 2.9 (with taking into account the definition of  $r$ ).

### 3. MadX model

The following scripts are used (download here [anybb.madx](#)):

- (1) A thin-element ring seqb1thin containing marker IP and a kicked orbit in  $x$  plane at a location  $s_{BB}$  sbb
- (2) Extract weak-beam parameters at l.r. location
- (3) Choose strong beam parameters
- (4) Track to create two dynaptune files w/o and with BB
- (5) Compute  $\Delta Q_{x,y}$

**Code MadX 1 Load thin-element weak-beam ring seqb1thin containing marker IP and kicked orbit**

```

1 call, file = "./seqb1thin";
2 nrj = 700;
3 bg = nrj/pmass;
4 en = 2.5e-06*200; ! changes
5 epsx = en/bg;
6 epsy = en/bg;
7 any=0.075;
8 Nb=1000*1.E11; !only changes ksi (and scales plots)
9 rp = 1.534698E-18;
10 ksi= (Nb*rp)/(4*Pi*en);
11 Beam, particle = proton, sequence=lhcb1, energy = nrj,
12         sigt=any,bv = +1, npart=Nb, sige= 1.1e-4,
13         ex=epsx, ey=epsy;

```

**Code MadX 2 Extract weak-beam parameters**

```

1 create,table=bbpar,column=sepx,dx,sigxstr,sigystr,sigxwk,sigywk,r,xstr,xwk;
2 sbb=-4; ! lr bb location on left of IP
3 bbmk:marker;
4 seqedit,sequence=lhcb1;
5 install,element=bbmk,at=sbb,from=IP;
6 endedit;
7 use,sequence=lhcb1;

```

```

8 twiss;
9 sigxwk = sqrt(table(twiss,bbmk,sig11))*1000;
10 sigywk = sqrt(table(twiss,bbmk,sig33))*1000;
11 xwk = table(twiss,bbmk,X)*1000;
12 ywk = table(twiss,bbmk,Y)*1000; !assumed zero
13 };

```

Lines 9 to end extract the weak-beam parameters: sigmas:  $\sigma_x^{\text{wk}}$   $\sigma_y^{\text{wk}}$  and orbit excursion  $x^{\text{wk}}$  at location  $s_{\text{bb}}$ . These are saved in table bbpar created on line 1.

**Code MadX 3** choose strong beam parameters at the l.r.

```

1 ! exact asymmetry weak-strong
2 !sigxstr=sigywk;
3 !sigystr=sigxwk;
4 ! arbitrary sigma strong
5 sigxstr=1.2;
6 sigystr=2.1;
7
8 xstr=15; ! XMA parameter in beambeam
9 r=sigystr/sigxstr;
10
11 ! full separation = Dx in Hamiltonian
12 sepx=xwk-xstr;
13 dx=sepx/sigxstr;
14 value,sepx,dx,r;
15 fill,table=bbpar;
16
17 ON_BB=1; ! flag to switch off beam beam
18
19 bb_any : beambeam,
20 sigx = sigxstr/1000,
21 sigy = sigystr/1000,
22 xma = xstr/1000,
23 yma = 0,
24 charge:= ON_BB;
25
26 seqedit,sequence=lhcb1;
27 install,element=bb_any,at=sbb,from=IP;
28 endedit;
29
30 write,table=bbpar,file=bbpar.dat;

```

Note the delayed assignment := on line 19.

Code *MadX 4* tracking creates two dynaptune files: w/o beam-beam in dir temp.0 and with beam-beam in dir temp.1

```
1  
2 use,sequence=lhcb1;  
3 twiss, sequence=lhcb1,save,file=twiss_b1_bef_track;  
4  
5 ! no bb  
6 ON_BB=0;  
7 use,sequence=lhcb1;  
8 call,file="./MAKEFOOTPRINT.madx";  
9 exec, MAKEFOOTPRINT($ON_BB);  
10  
11 !with bb  
12 ON_BB=1;  
13 use,sequence=lhcb1;  
14 call,file="./MAKEFOOTPRINT.madx";  
15 exec, MAKEFOOTPRINT($ON_BB);
```

## Code MadX 5 MAKEFOOTPRINT

```
1 MAKEFOOTPRINT(nam): macro = {
2   system, "rm -rf temp\_nam";
3   system, "mkdir temp\_nam";
4   set,      format="30.15f";
5   nturn=2000;
6   mtot=2;
7   amin=1;
8   amax=-sepx/sigxwk;
9   dn= amax/5;
10  small=0.05;
11  big=sqrt(1.-small^2);
12  delete,table=dynaptune;
13  delete,table=aini;
14  create,table=aini,column=xs,ys;
15  dpp=0;
16  track,deltap=dpp,dump;
17  n=amin;
18  m=0;
19  while (n <= amax)
20  {
21  angle = 90/mtot*m*pi/180;
22  if (m == 0) {xs=n*big; ys=n*small;}
23  elseif (m == mtot) {xs=n*small; ys=n*big;}
24  else
25  {
26  xs=n*cos(angle);
27  ys=n*sin(angle);
28  }
29  value,xs,ys;
30  fill,table=aini;
31  start,fx=xs,fy=ys;
32  m=m+1;
33  if (m == mtot + 1 ) { m=0; n=n+dn;}
34  }
35  run,turns=nturn;
36  dynap,fastune,turns=nturn;
37  endtrack;
38  write,table=aini,file="aini.dat";
39  write,table=dynaptune,file="temp\_nam/dynaptune";
40  };
```

## Code Mathematica 6

```

1 A = sigxwk /sigystr;
2 B = sigywk/sigxstr;
3 - 2 ksi {
4   A^2 DXC00 [dx,0,r,A ax,B ay]/ax
5   ,
6   B^2 DYC00 [dx,0,r,A ax,B ay]/ay
7   };

```

The  $a_x, a_y$  are taken from file `aini.dat`.

The parameters  $dx, sigxstr, sigystr, sigxwk, sigywk, r$  are taken from the table saved in file `bbpar.dat`.

The DXC00, DYC00 are defined in [1].

## 4. Results

Parameters for the example shown on Figures 1, 2, 3 and 4 (all lengths in mm) :

$sepx = -27$   $sigxstr = 1.2$   $sigystr = 2.1$   $sigxwk = 2.57$   $sigywk = 2.55$   $dx = -22.5$   $r = 1.75$

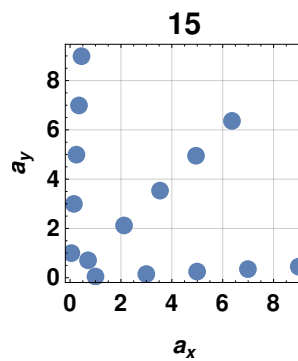


Figure 1. Weak-beam particle amplitudes.

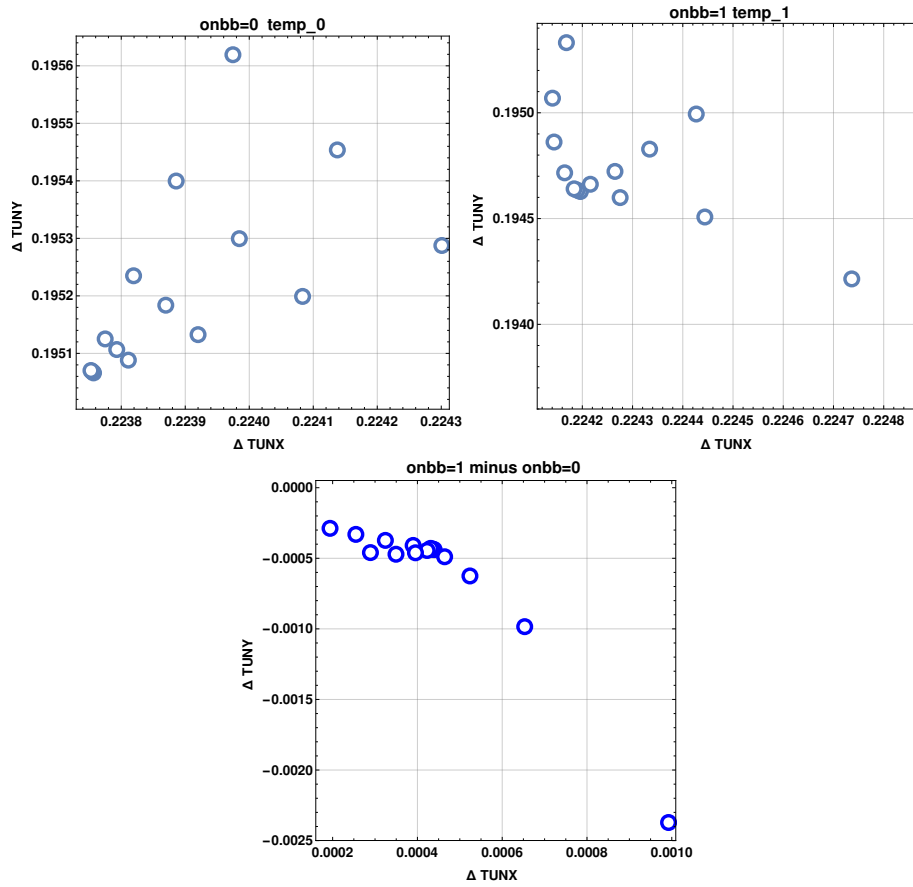


Figure 2. MadX detunings are the difference between the onbb=1 and onbb=0 detunings

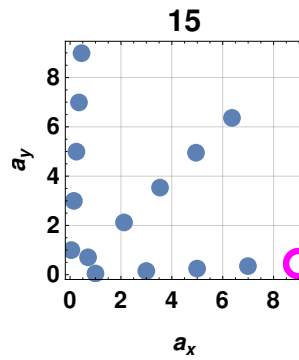


Figure 3. Divergent amplitude point near str beam core – magenta



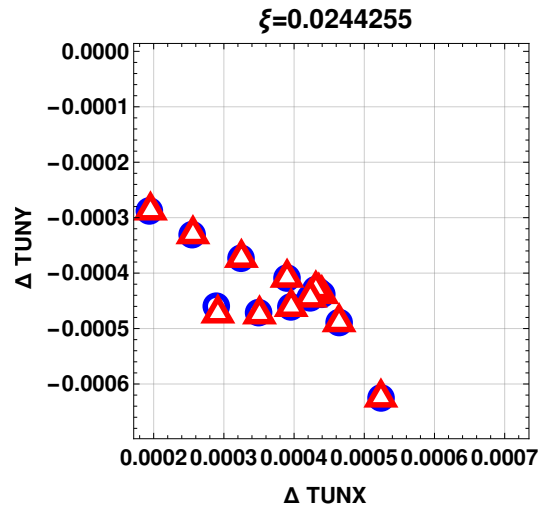


Figure 4. MadX detuning (blue) and analytic, Eqn 1.5 (red).

## References

- [1] D. Kaltchev, *Beam-beam detuning formula and its agreement with MadX. Individual collisions*, TRI-BN 2214, June 19, 2022, [PDF](#)
- [2] D. Kaltchev, *Fourier Coefficients of Long-Range Beam-Beam Hamiltonian via Two-Dimensional Bessel functions* Proc. of IPAC 2018, Vancouver BC, Canada